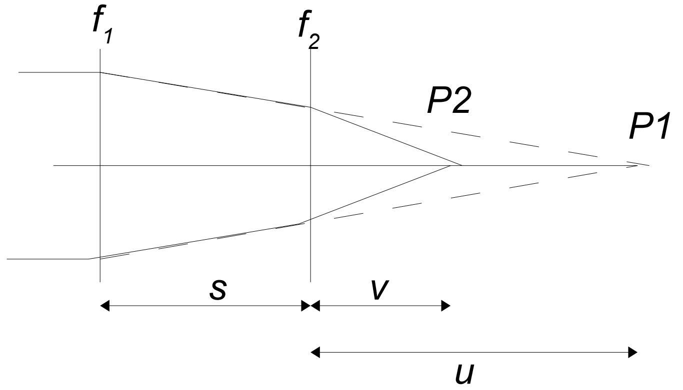
The Optics of Camera Lenses

The basic formula

Consider two converging lenses of focal lengths f_1 and f_2 separated by a distance *s*.



The front lens focuses an image of an object at infinity at the point *P1*. This image is focused by the rear lens at the point *P2*.

When using the lens formula $1/f = 1/u + 1/v$ we must remember that, as far as the rear lens is concerned, *P1* is what is known as a *virtual object* and, if we are to regard all the distances shown in the diagram as positive, we must put a minus sign before *u*. This gives us:

$$\frac{1}{f2} = \frac{1}{-u} + \frac{1}{v} \quad (1)$$

hence

$$v = \frac{f_2 u}{f_2 + u} \quad (2)$$

where

$$u = f_1 - s \quad (3)$$

In theory, any combination of lenses may be replaced by a single lens with a certain focal length. What we want to know now is, what is the focal length of this combination of lenses? This is important because it is the focal length of the lens which determines how big the image will be. The longer the focal length of a lens, the larger the image will be on the film. To see this, consider the size of the image of the Moon on the sensor in a camera. The Moon subtends an angle of half a degree or 0.009 radians. When it is focused by a standard lens of focal length 50 mm, it forms an image 50×0.009 = 0.45 mm across. If you fit a telephoto lens with a focal length of 120 mm on the camera, the image will be 120×0.009 = 1.1 mm across. In general, the diameter the image of an object which subtends an angle *α* at the camera lens will be *αf*. Conversely, if the diameter of the image is *d*, the focal length will be *d/α*.

The formula we have derived tells us where the image will come to a focus but it does not tell us how big the image will be. Fortunately it is not difficult to work it out. The front lens will create and image whose diameter is αf_1. This image is 'magnified' by the rear lens by a factor of *v/u*. (I say 'magnified' because in this instance, the final image is even smaller.). So the diameter of the final image will be $\alpha f_1 \times \frac{v}{u}$ and hence the effective focal length of the combination of the two lenses will be $f_1 \times \frac{v}{u}$. Substituting our values of *u* and *v* we get

$$f_{comb} = \frac{f_1 f_2}{f_2 + f_1 - s} \quad (4)$$

The is the fundamental formula for two lenses in combination. We can, however, simplify it considerably by using dioptres instead of focal lengths.

The power of a lens in dioptres F is one over the focal length in metres (or 100 over the focal length in cm). i.e. $F = 1/f$. Plugging this in we get:

$$F_{comb} = F_1 + F_2 - sF_1F_2 \qquad (5)$$

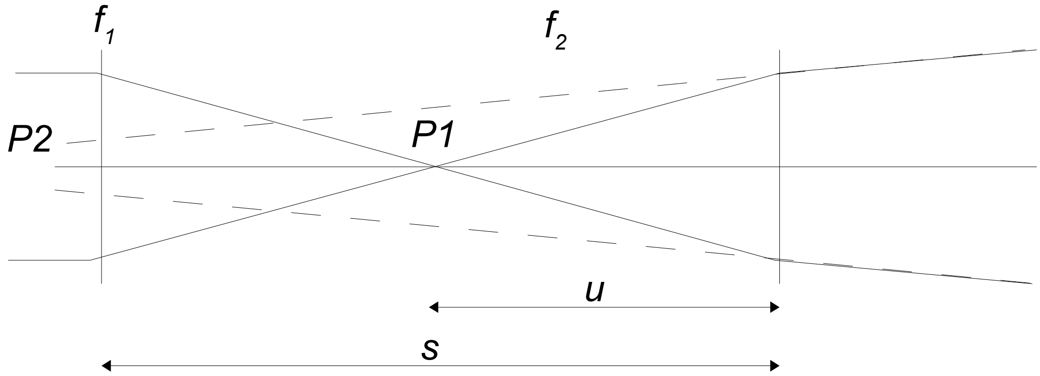
Lets put in a few figures to see how it works. Suppose we have two 10 cm (10D) lenses. If we place them in contact, $s = 0$ and the powers simply add together.

If we place them 5 cm (0.05 m) apart then the effective power of the combination is $10 + 10 - 0.05\times10\times10 = 15$D and its effective focal length will be 6.7 cm. The further apart the lenses are,the smaller the effective power and the longer its focal length will be.

Some special situations

What happens if separation s is equal to the focal length of the front lens? The formula gives us $F_{comb} = 10$D. This makes sense as the rear lens now plays no part in altering the position or size of the image. (It also gets in the way of the image so the arrangement is pretty useless!)

But what if s is greater than f_1? Say $s = 19$ cm – what then? The formula tells us that the combination will have a power of 1D and a massive focal length of 100 cm. The reason for this is best explained with a diagram:



It is now apparent that the rear lens is creating a large but *virtual* image at P_2 so this arrangement is no use as a telephoto lens. It is, however, the basis for use as an astronomical telescope (in which case the separation s is equal to the sum of the two focal lengths and f_2 is much shorter than f_1.)

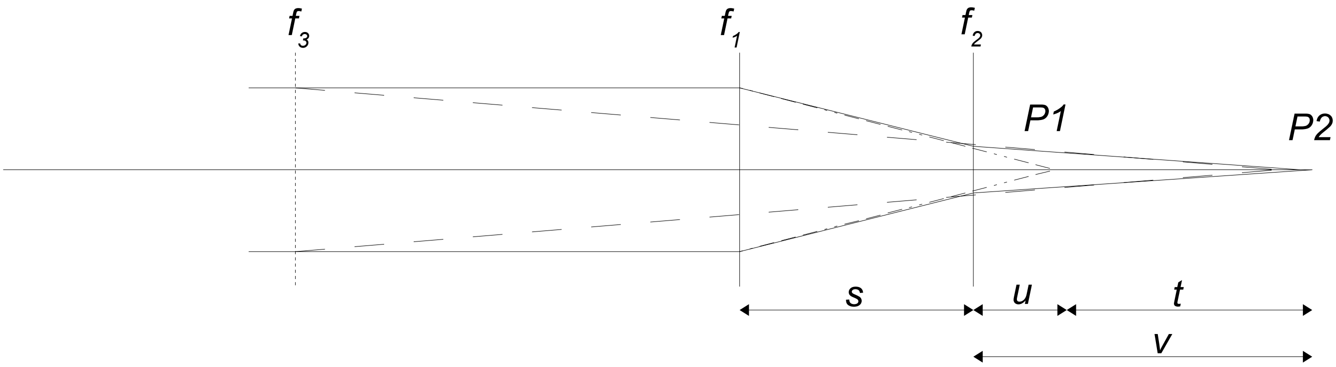
What happens if s is even greater than the sum of the two focal lengths? e.g. if $s = 21$ cm then $F_{comb} = -1$D. The minus sign is not significant[1]. This combination will still produce a real image which could be focused on a sensor, and it would be as large as the image produced by 1000 mm (1D) lens – but it would still not be much use as a way of constructing a lens with a long focal length because the length of the combination would be even longer.

The telephoto lens

So how do you construct very long focus lenses such as those used by journalists and wild life photographers but using shorter lenses? The answer is to use a diverging lens as the rear lens. The formula will still work as before, but we must use a negative value for the power of this lens. For

1 The reason why the sign is not significant is that we have derived the formula using the diameter of the final image without reference to whether it is real or virtual. The formula gives the correct power and focal length of the combination but we have to work out what kind of image it produces separately.

example, suppose F_1 = 20D, F_2 = –75D and s = 0.04 m. Parallel light from a distant object we be focused in the following way:



The front lens focuses the light on to its principal focal point, P_1, but before the light gets there, it is diverged slightly by the rear lens, causing it to come to a focus at P_2. It is as if the light had actually been focused by a single lens f_3 which is positioned way out in front of the primary lens. In this example, v turns out to be 4 cm and f_3 = 20 cm. This means that we have packed a 200 mm focal length lens into a package which is only 80 mm long. (Alternatively, using equation (5) we get F_{comb} = 20 – 75 + 0.04×20×75 = 5D i.e. a focal length of 200 mm.)

Barlow lenses

A Barlow lens is a diverging lens which is inserted between a primary lens and the camera body. It effectively turns a standard lens into a telephoto lens. Usually Barlow lenses are specified as being ×2 or ×3. If we consider the diagram above, it is clear that the addition of the Barlow lens must push the primary lens forward by a distance $t = v - u$. (i.e. t is the width of the lens mount.) In the case of a ×n lens, $v = nu$. A little bit of algebra reveals that the focal length of the diverging lens has to be $f = -\frac{n}{(n-1)^2} \times t$ so a ×2 Barlow lens has a focal length of $-2t$ and a ×3 lens would require a focal length of $-\frac{3}{4}t$.

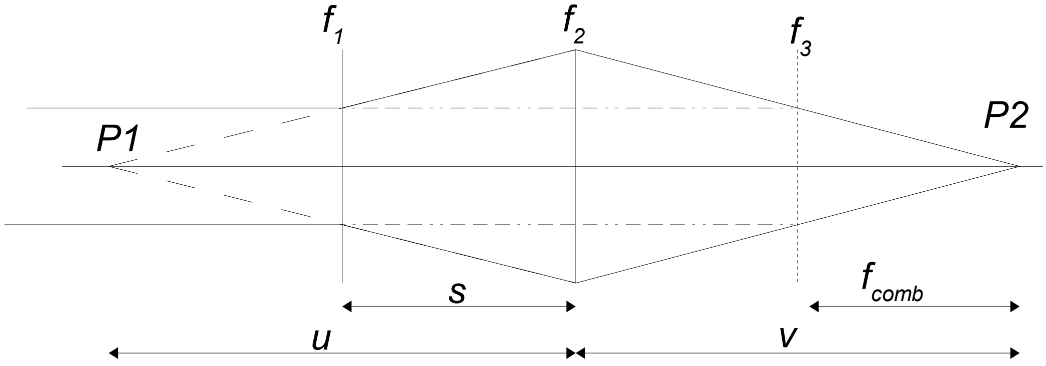
Zoom lenses

Whereas a standard telephoto lens will have fixed lenses, it is easy to see from equation (5) that altering s (and moving both lenses to maintain the principal focal point in the same place) will change the effective focal length of the lens. Using the figures in our example, when s = 0, the effective power of the combination is zero and no image is formed. As we have seen, when s = 0.05 m the focal length is 200 mm and when s = 10 cm the diverging lens has no effect and the effective focal length is equal to that of the primary lens i.e. 100 mm. The potential range of focal lengths which are available with these lenses is therefore 100 mm to infinity – but in practice, if we restrict the range to something like 2.5 cm <= s <= 6.7 cm then the range goes from 150 mm to 400 mm – quite a useful range for a 35 mm camera.

Modern 'bridge' cameras (which 'bridge' the gap between point-and-shoot cameras and digital SLR's) are often equipped with quite amazing zoom lenses with focal lengths which can sweep from as little as 4.3 mm to 213 mm or more. Although these lenses must use exactly the same principle as that outlined above, quite how they achieve such a wide range of focal lengths is a mystery. Almost certainly they use at least three movable lenses (or more likely, groups of lenses) whose motions are choreographed by precisely calculated tracks engraved in the mechanism. The fact that they can do this and still produce images of an acceptable (if not brilliant) quality at a price which ordinary people can afford is little short of miraculous.

Wide angle lenses

Once you have appreciated the principle behind a telephoto lens, it should not be too difficult to see how wide angle lenses work. Like the telephoto lens, a wide angle lens (often called a *retrofocus* lens) has two elements. This time, however, the front lens is diverging and the rear one is converging. The optical system looks like this:



This combination of lenses effectively acts like a converging lens at f_3 with a focal length of f_{comb}.

The obvious question to ask is – why bother to use two lenses when a single lens at f_3 would be just as good. One answer is that in a Single Lens Reflex camera, there must be sufficient space between the back of the lens and the sensor (the *back focus*) to allow the mirror to swing out of the way. Another answer is that, having more than one lens gives the designer more flexibility to minimise the various distortions and aberrations which single lenses are prone to.

Suppose the front lens has a power of –20D and the rear lens +50D with s = 2 cm. Using equation (5) we find that the power of the combination is 50D and has an effective focal length of 20 mm. This does not sound like much of an improvement because, surprisingly, the addition of the diverging lens has not changed the focal length of the overall lens at all. This is because, with the figures I have suggested, the diverging lens is actually positioned at the front principal focus of the primary lens. In fact it is true in general that a lens placed here (whether diverging or converging) will not change the overall power of the lens at all.

So what is the point of this arrangement? Without the diverging lens, the back focus of the lens is equal to its focal length of 2 cm. With it, the back focus increases to 2.8 cm[2].

Supplementary wide angle lenses

A supplementary wide angle lens is a diverging lens which is screwed on to the front of an existing lens. Paradoxically it makes the effective focal length of the primary lens shorter. If we rewrite the basic equation as:

$$F_{comb} = F_1(1 - sF_2) + F_2 \tag{6}$$

then, bearing in mind that F_1 is negative (because it represents a diverging lens) and that we want to effectively *increase* the power of our lens (i.e. making its focal length shorter), it is clear that the term in brackets must also be *negative*. This means that sF_2 must be *greater than 1*. In other words, s must be greater than f_2. This poses a bit of a problem because the typical focal length for a standard 35 mm SLR camera is 50 mm (20D). Suppose we wish to shorten this to 35 mm (28.6D). We now have:

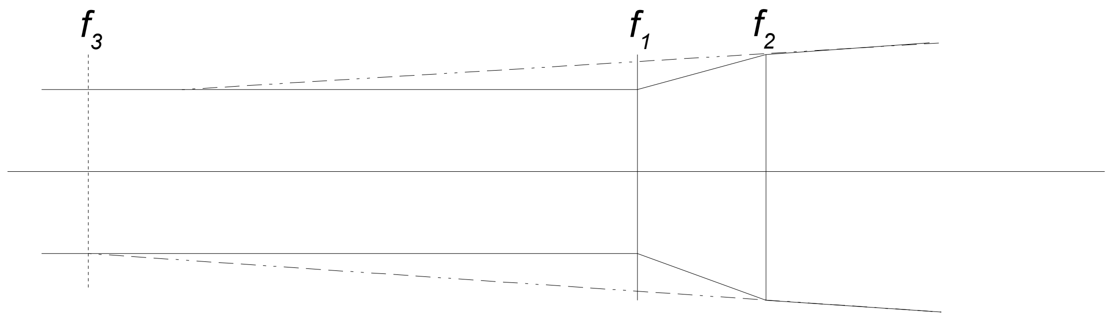
2 This assumes that the lens is infinitely thin. In practice, what I have been referring to as a 'lens' will in fact be a group of several lenses fixed together in a group which will itself be a combination of converging and diverging lenses.

$$\begin{aligned} 28.6 &= F_1(1 - 20\,s) + 20 \\ F_1(1 - 20\,s) &= 8.6 \end{aligned} \tag{7}$$

In order to make the term in brackets negative *s* must be at least 0.05 m. Suppose we decide on a value of 0.075 m (75 mm) for the value of *s*. The term in brackets evaluates to –0.5 and F_2 turns out to be –17.2D. This is a fairly chunky piece of glass to screw on to the front of a camera!

There is another consideration to take into account. The supplementary lens f_1 creates a virtual image at P_1. This image must be sufficiently far away from the primary lens f_2 in order for the camera to focus properly and form an image on the sensor at P_2. Since = (–) 17.2D lens has a focal length of (–) 58 mm, the virtual image at P_2 will only be 58 + 75 = 133 mm in front of the primary lens which must therefore be able to focus on objects which are that close.

To some extent, both of these problems can be alleviated by clever design of the supplementary lens. Looking at the diagram of the telephoto lens on page 3 you will see that the effective position of the lens is actually way out in front of the actual lenses. We can perform the same sort of trick to create a diverging lens which is effectively positioned way out in front of itself in the following way:



It consists of a strong diverging lens followed by a weaker converging one.

Yet another problem relates to the necessity of gathering light from a very wide angle without 'vignetting' or distortion.

Finally, we have the problem of knowing how to specify or describe the effects of a supplementary lens. For example, I have a lens for my Canon 1000D SLR which is marked ×0.5. This is what it looks like:

A ×0.5 Supplementary Wide Angle Lens

When attached to a standard 18-55 mm lens set to its widest angle (i.e. 18 mm) it increases the field of view from 65° to 80°. An image printed on the box strongly suggests that the lens will almost double the field of view but it is perfectly clear just looking through the lens that it could never achieve that kind of 'minification'.

Now 80/65 = 1.25 and 1.25^2 is approximately equal to 1.5. Could it be that the manufacturers justify the figure ×0.5 on the basis that the *area* of the coverage is increased by approximately 50%? If so they are stretching logic well beyond breaking point.

Another possibility is that the manufacturers are wishing to compare the effect of their lens with that of a magnifying glass which is often specified as being ×5 or ×10. The formula which relates the 'magnifying power' of a magnifying glass to its optical power in dioptres is

$$\textit{Magnifying power} \;=\; D_{eye}F \;+\; 1 \quad \text{or} \quad \frac{D_{eye}}{f} \;+\; 1 \tag{8}$$

where D_{eye} is the closest distance at which the normal human eye can focus and see clearly. This distance is taken to be 0.25 m. For example, a 10D lens will have a magnifying power of 3.5. Conversely, a ×10 lens will have a power of 36D.

Now the above formula can only be usefully used to specify the magnifying power of a converging lens. It makes no sense at all to apply the formula to a supplementary wide angle lens – after all, what has a camera got to to with the closest distance of distinct vision of the human eye? On the other hand, it is probably too much to expect advertisers to fuss with details like that so let us blindly apply the formula and see what we get. Putting *Magnifying power* = 0.5 we find that the optical power of the 'minifying glass' would be –2D. Well, at least it turned out to be negative!

It is impossible to determine *s* accurately but using the technique described above it could be as much as 10 cm or even more. This would mean that our 18 mm (55.5D) primary lens would become –2 + 55.5 + 0.1×2×55.5 = –2 + 55.5 + 11.1 = 64.6D – an increase of 16%. In order to achieve the measured 25% increase, *s* would have to be about 15 cm. This is perfectly feasible.

The truth is that, unlike the Barlow lens, there is no simple way of specifying exactly what the effect of the lens will be on any particular primary lens. You could, for example, say that the lens has a power of –2D, but how much this will change the focal length of your primary lens depends on *s* which is largely unknown so the information is pretty useless. All you can do is ignore what the manufactures claim and try it out with your particular camera.

Happy snapping!

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